

# APPENDIX — One Ant is Not Enough

Benjamin Doerr, Daniel Johannsen, Ching Hoo Tang

Max-Planck-Institut für Informatik  
Saarbrücken, Germany

**Abstract.** Recent results of different authors suggest that already quite simple ant colony optimization (ACO) systems comprising a single ant can be useful to efficiently optimize pseudo-boolean functions. In addition, there is the hope that via such simple systems more understanding of the in practice highly successful ACO systems can be gained.

We undertake a rigorous experimental analysis of the two most studied single ant ACO systems. By tracking the behavior of the underlying random processes describing the ant walks throughout the whole run of the system (and not just regarding the resulting optimization time), we gain more insight into these systems, which puts some doubt on previous optimism concerning single ant ACO systems. A main finding is that in those cases where the single ant ACO system performs well, it basically simulates the well-known and much simpler (1+1) evolutionary algorithm.

This appendix provides the test datas that had to be omitted from the paper for reasons of space.

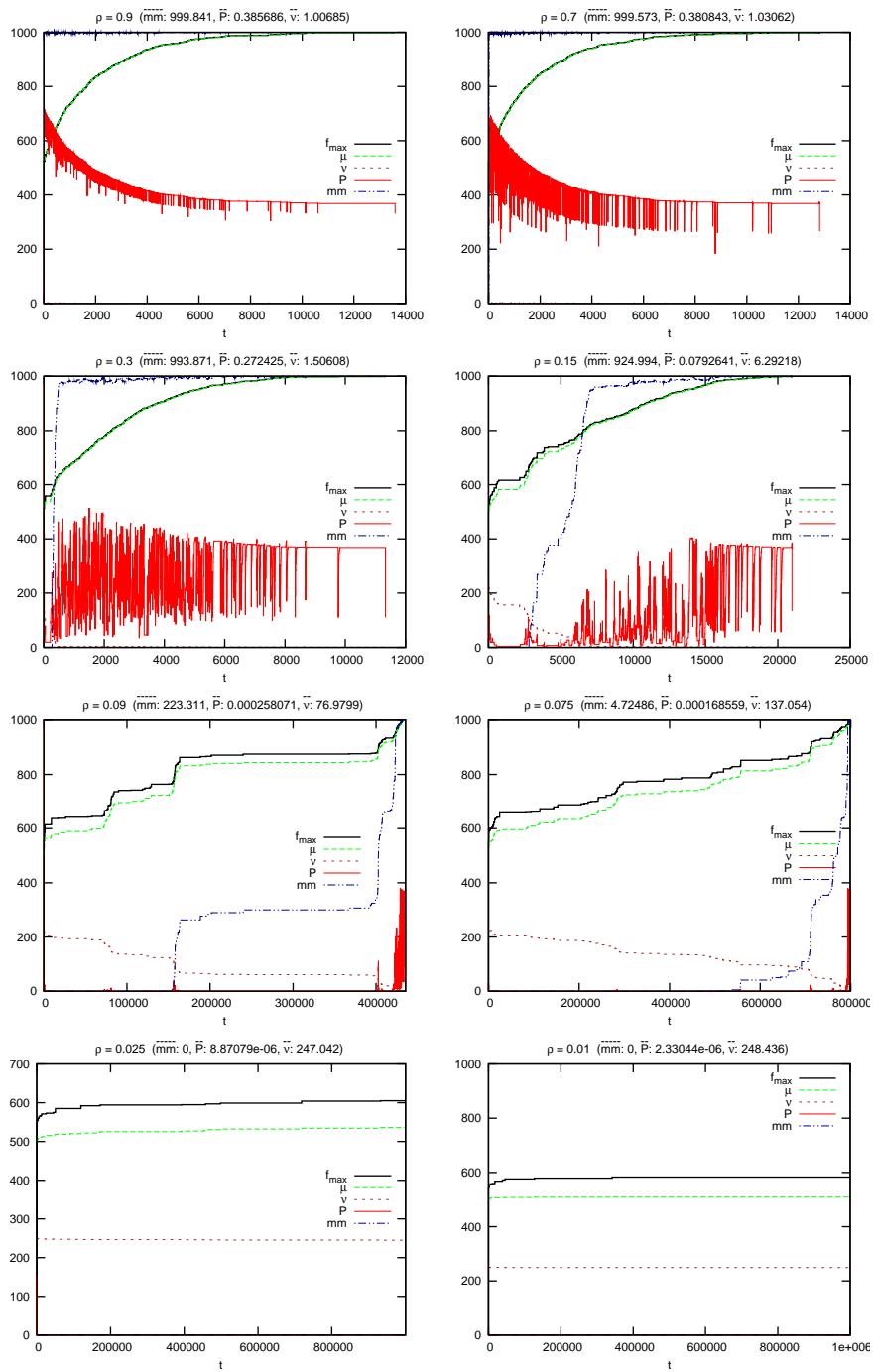
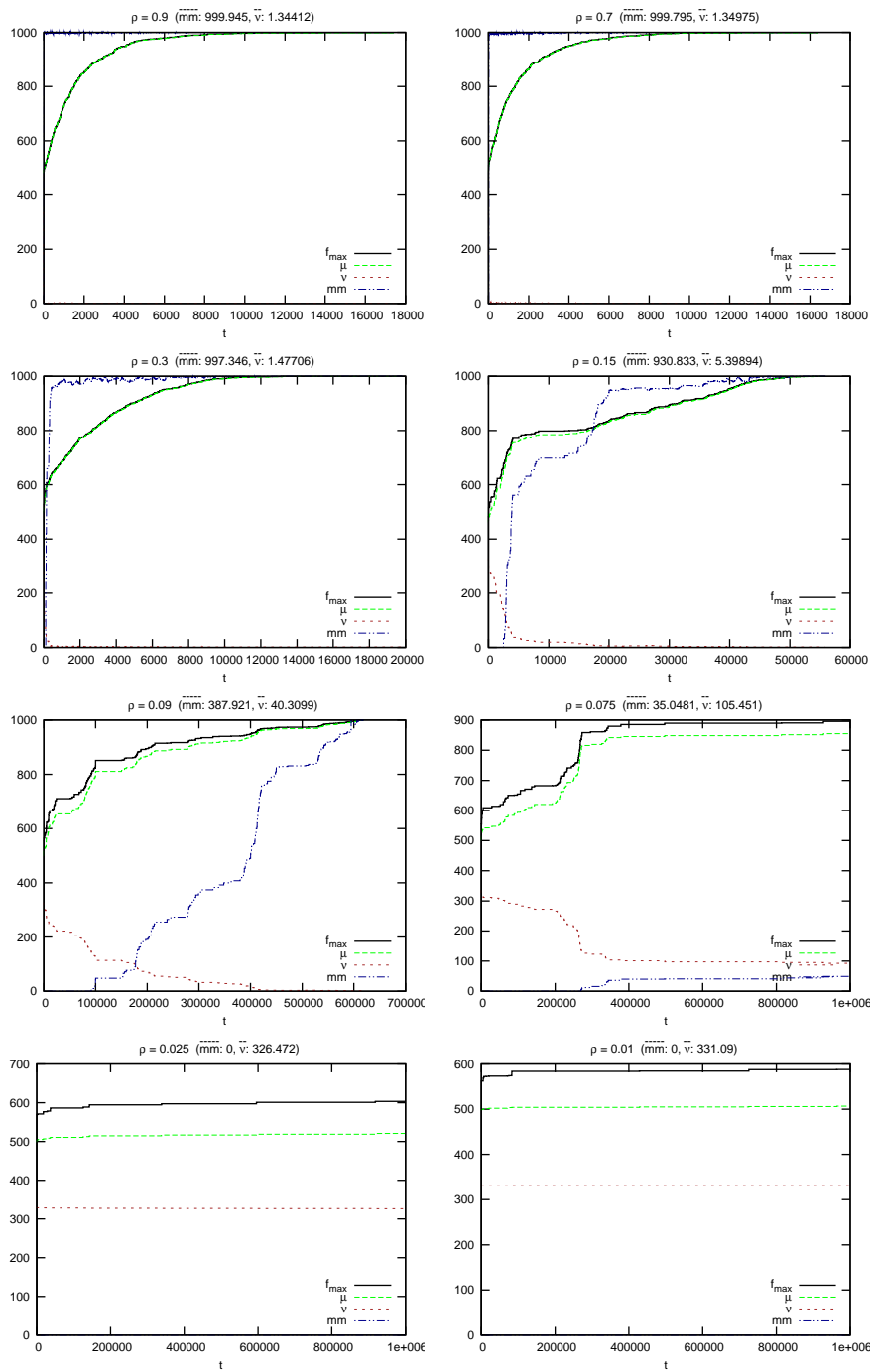


Fig. 1. Eight typical runs of 1-Ant on ONEMAX for different values of  $\rho$ .



**Fig. 2.** Eight typical runs of 1-Ant on random linear functions for different values of  $\rho$ .

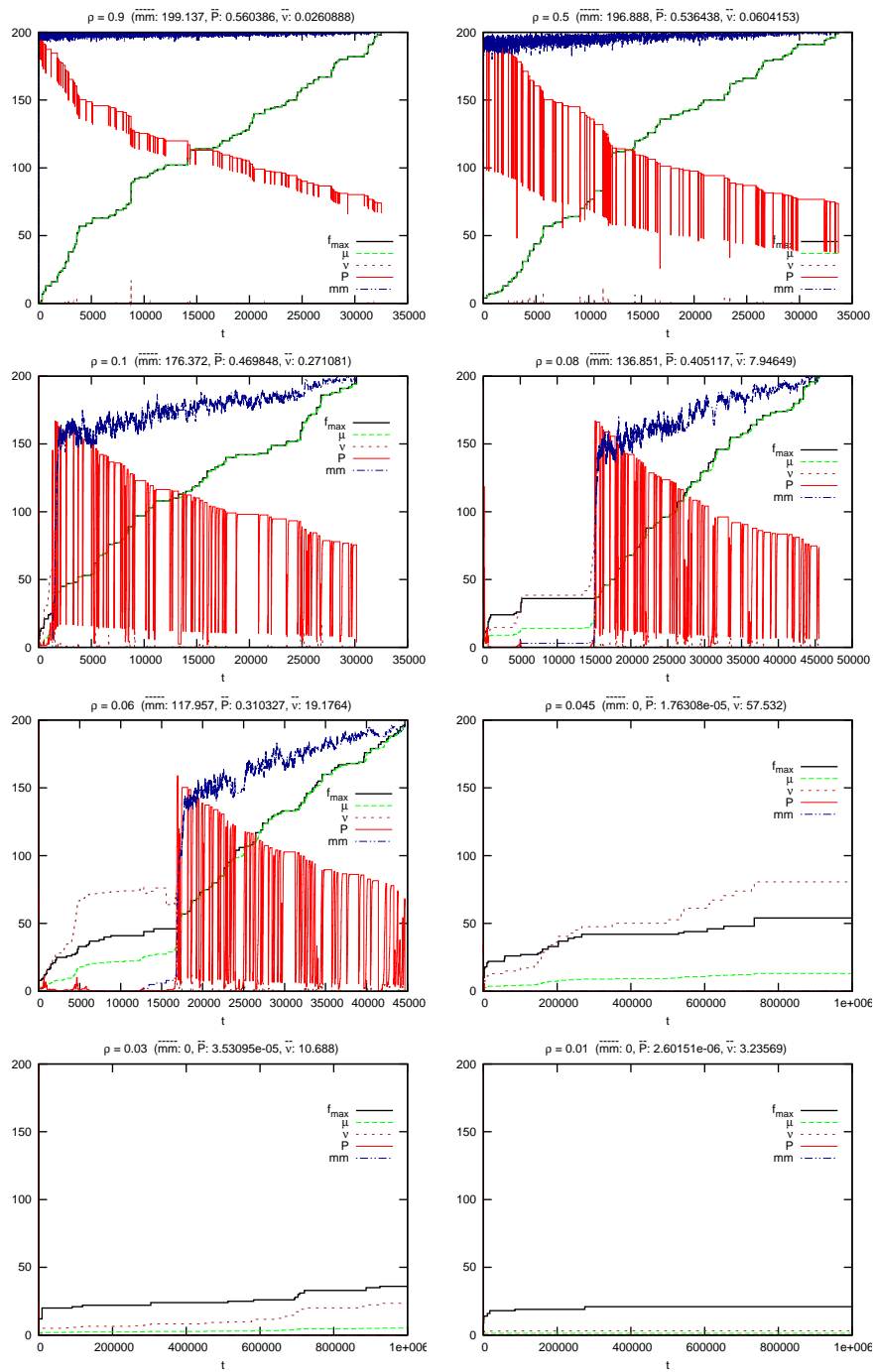


Fig. 3. Eight typical runs of 1-Ant on LEADINGONES for different values of  $\rho$ .

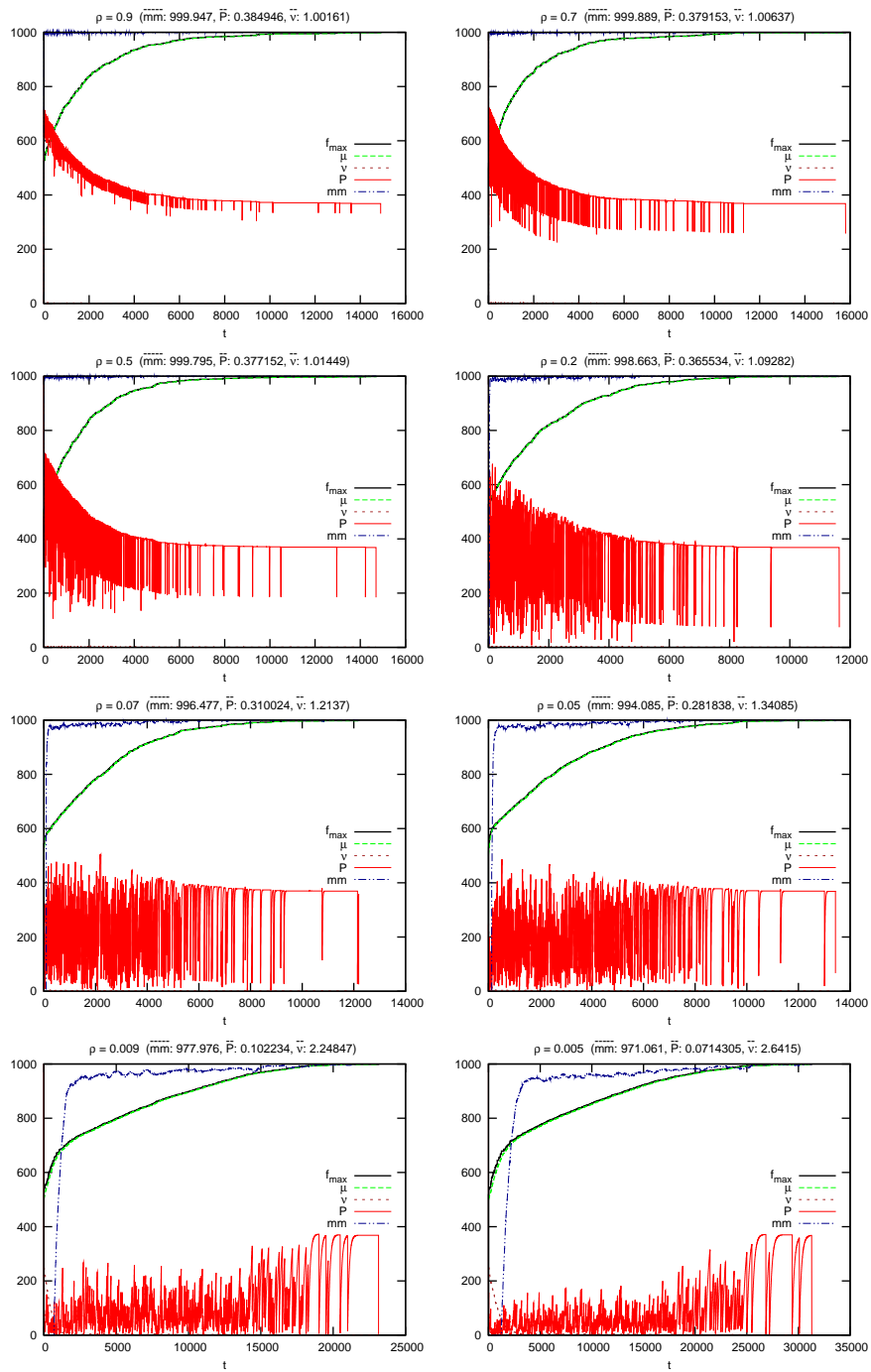
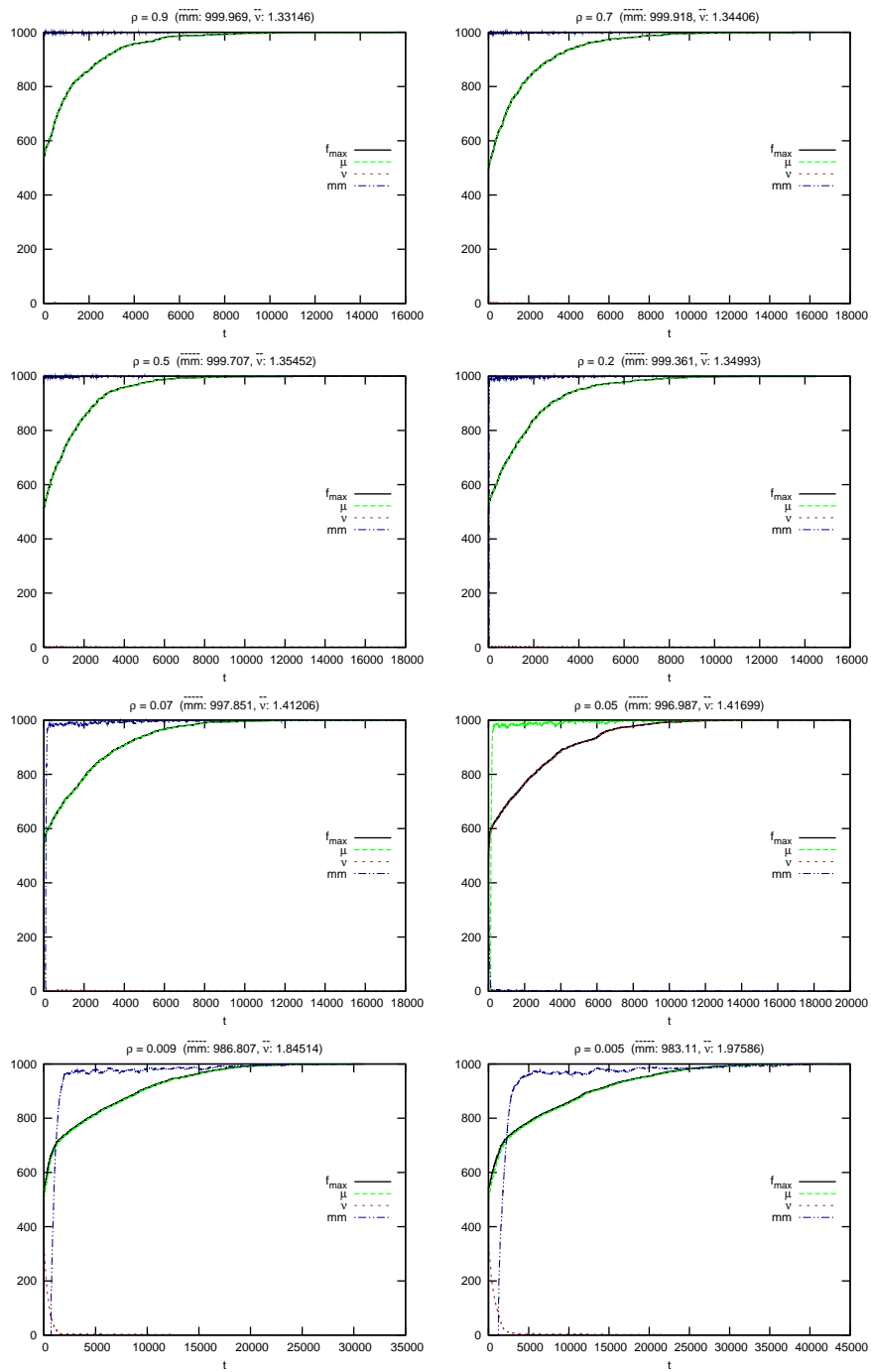


Fig. 4. Eight typical runs of the MMAS on ONEMAX for different values of  $\rho$ .



**Fig. 5.** Eight typical runs of the MMAS on random linear functions for different values of  $\rho$ .

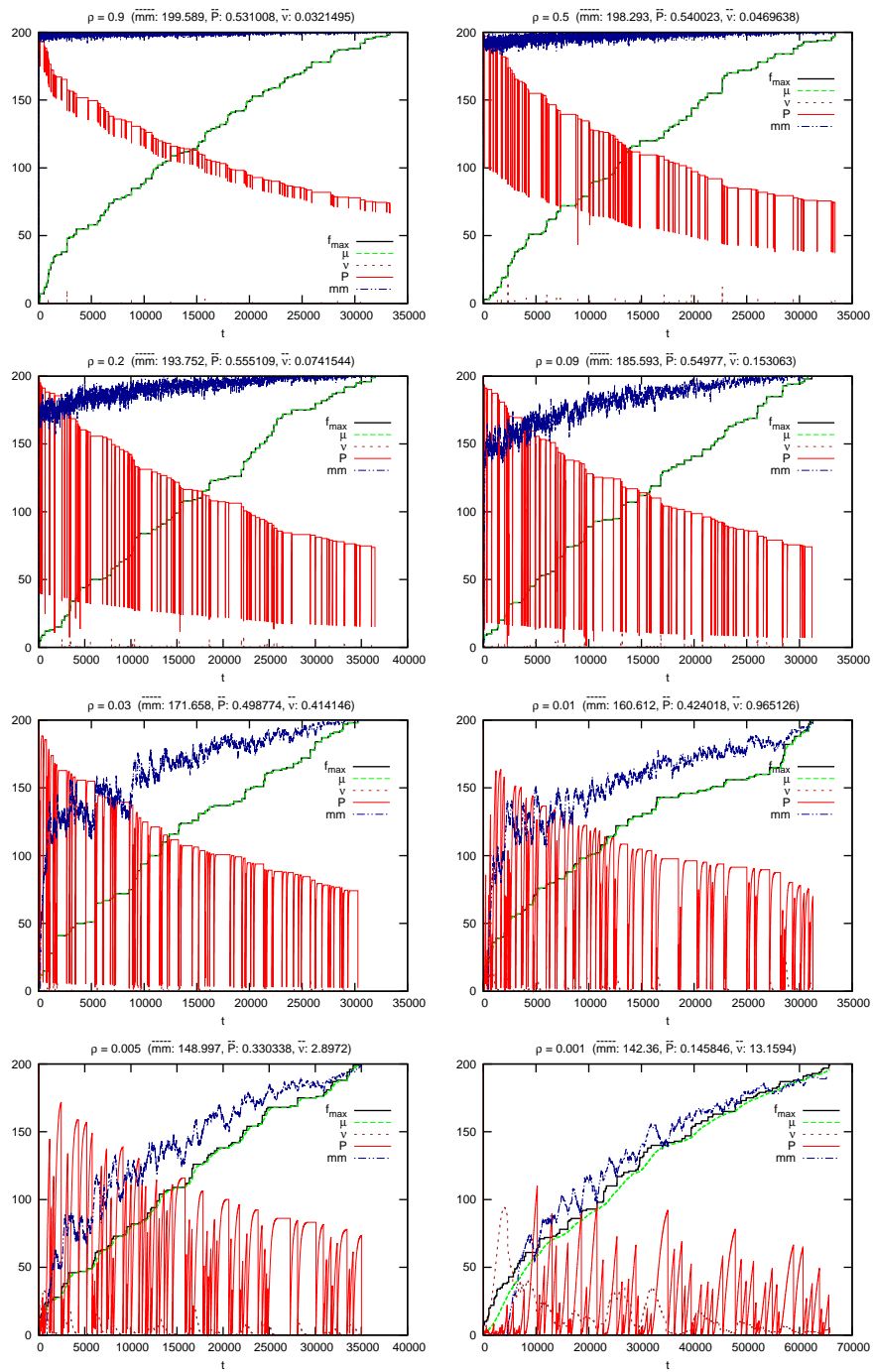


Fig. 6. Eight typical runs of the MMAS on LEADINGONES for different values of  $\rho$ .

$\rho$	ONEMAX						random linear			
	$T$	$\overline{mm}$	$\overline{P}$	$\overline{\nu}$	$f_{\max}^{(T)}$	$P^{(T)}$	$T$	$\overline{mm}$	$\overline{\nu}$	$f_{\max}^{(T)}$
1.0	16459	1000.00	0.38	0.999			17459	1000.00	1.314	
0.95	18314	999.90	0.38	1.002			17357	999.92	1.320	
0.9	15173	999.86	0.38	1.006			17403	999.94	1.344	
0.8	13059	999.65	0.38	1.019			17015	999.84	1.332	
0.7	12570	999.51	0.38	1.035			17497	999.79	1.346	
0.6	12167	999.37	0.37	1.049			15614	999.65	1.343	
0.5	11352	998.87	0.36	1.096			16406	999.45	1.340	
0.4	11247	998.11	0.35	1.152			18776	999.22	1.392	
0.35	11205	997.07	0.33	1.233			17561	998.67	1.425	
0.3	11353	995.75	0.28	1.344			19524	997.02	1.520	
0.25	12599	993.07	0.25	1.553			23423	994.56	1.663	
0.2	15871	988.57	0.19	1.889			31572	976.73	3.091	
0.15	30949	851.32	0.07	13.151			61404	899.98	8.905	
0.1	165751	373.26	$1.31 \cdot 10^{-3}$	63.695			418633	393.22	55.167	
0.095	194638	268.84	$1.11 \cdot 10^{-3}$	79.577			809135	367.07	49.710	
0.09	317326	170.50	$7.41 \cdot 10^{-4}$	95.782			822138	302.06	80.384	
0.085	404953	135.98	$4.35 \cdot 10^{-4}$	99.465			972004	338.75	61.677	
0.08	443073	173.45	$6.88 \cdot 10^{-4}$	95.193			1000000	30.90	156.999	885.261
0.075	845805	112.04	$1.92 \cdot 10^{-4}$	130.172			1000000	83.39	129.252	883.966
0.07	950354	28.51	$9.90 \cdot 10^{-5}$	135.543			1000000	3.92	223.849	818.387
0.06	1000000	0.79	$8.37 \cdot 10^{-5}$	188.428	791	$1.94 \cdot 10^{-5}$	1000000	0.00	254.443	751.683
0.05	1000000	0.00	$2.58 \cdot 10^{-5}$	218.335	687	$6.31 \cdot 10^{-6}$	1000000	0.00	302.602	652.275
0.025	1000000	0.00	$5.82 \cdot 10^{-6}$	246.326	604	$4.50 \cdot 10^{-6}$	1000000	0.00	326.273	609.832
0.01	1000000	0.00	$4.08 \cdot 10^{-6}$	248.496	581	$2.08 \cdot 10^{-6}$	1000000	0.00	331.083	593.756

**Table 1.** Results for 1-Ant optimizing the fitness functions ONEMAX and random linear functions for different values of  $\rho$ . All numbers are the averages over 10 runs. The table demonstrates an (1+1) EA-typical behavior for  $\rho \geq 0.2$ , whereas for  $\rho \leq 0.1$  the ant system gets stuck, i. e., has a very small probability of finding an acceptable solution.



$\rho$	$T$	$\overline{mm}$	$\overline{P}$	$\overline{\nu}$	$f_{\max}^{(T)}$	$P^{(T)}$
1.0	33428	200.00	0.55	0.028		
0.9	32985	199.14	0.56	0.035		
0.8	32981	198.64	0.56	0.034		
0.7	37106	198.39	0.52	0.038		
0.6	32758	197.61	0.54	0.054		
0.5	33548	196.73	0.54	0.062		
0.4	30268	195.15	0.54	0.095		
0.3	34140	192.95	0.53	0.101		
0.2	36111	188.72	0.52	0.155		
0.1	33915	178.61	0.46	0.461		
0.095	36839	169.04	0.45	1.990		
0.09	38783	165.62	0.44	4.130		
0.085	78121	157.35	0.39	1.732		
0.08	46904	138.14	0.38	8.622		
0.075	58257	119.40	0.31	9.893		
0.07	321854	38.44	0.10	37.363		
0.065	315159	84.66	0.19	29.040		
0.06	473330	41.93	0.11	35.452		
0.055	573244	23.06	0.04	49.797		
0.05	614598	12.37	0.02	53.867		
0.045	1000000	1.53	$3.91 \cdot 10^{-5}$	55.475	52	$7.52 \cdot 10^{-6}$
0.04	1000000	0.00	$1.81 \cdot 10^{-5}$	27.606	41	$9.62 \cdot 10^{-6}$
0.035	1000000	0.00	$1.28 \cdot 10^{-5}$	23.976	39	$3.56 \cdot 10^{-6}$
0.03	1000000	0.00	$1.88 \cdot 10^{-5}$	14.571	34	$4.44 \cdot 10^{-6}$
0.025	1000000	0.00	$1.16 \cdot 10^{-5}$	9.743	30	$4.54 \cdot 10^{-6}$
0.02	1000000	0.00	$5.94 \cdot 10^{-6}$	5.928	27	$2.24 \cdot 10^{-6}$
0.015	1000000	0.00	$4.40 \cdot 10^{-6}$	3.781	24	$1.94 \cdot 10^{-6}$
0.01	1000000	0.00	$5.67 \cdot 10^{-6}$	3.180	23	$1.28 \cdot 10^{-6}$
0.009	1000000	0.00	$4.23 \cdot 10^{-6}$	3.157	22	$2.09 \cdot 10^{-6}$
0.008	1000000	0.00	$3.48 \cdot 10^{-6}$	3.065	23	$8.58 \cdot 10^{-7}$
0.001	1000000	0.00	$3.43 \cdot 10^{-6}$	2.087	21	$1.17 \cdot 10^{-6}$

**Table 2.** Results for 1-Ant optimizing the fitness function LEADINGONES for different values of  $\rho$ . All numbers are the averages over 10 runs.

$\rho$	ONEMAX				random linear		
	$T$	$\overline{mm}$	$\overline{P}$	$\overline{v}$	$T$	$\overline{mm}$	$\overline{v}$
1.0	16459	1000.00	0.38	0.999	18311	1000.00	1.336
0.95	17647	999.96	0.38	1.000	16696	999.97	1.338
0.9	14883	999.94	0.39	1.002	15874	999.97	1.332
0.85	17647	999.93	0.38	1.002	17516	999.96	1.343
0.8	14310	999.88	0.39	1.006	16069	999.94	1.302
0.75	14873	999.88	0.39	1.007	15663	999.94	1.343
0.7	15689	999.87	0.38	1.008	16272	999.92	1.345
0.65	15501	999.86	0.38	1.009	15974	999.91	1.336
0.6	13686	999.78	0.38	1.014	15411	999.88	1.318
0.55	15419	999.81	0.38	1.013	16937	999.89	1.327
0.5	13576	999.73	0.38	1.020	17508	999.89	1.323
0.45	13216	999.66	0.38	1.024	17116	999.87	1.327
0.4	14649	999.66	0.38	1.029	17517	999.84	1.333
0.35	11751	999.35	0.38	1.046	16004	999.73	1.349
0.3	12738	999.33	0.37	1.048	16241	999.68	1.340
0.25	11286	998.88	0.37	1.079	17130	999.62	1.342
0.2	12644	998.70	0.37	1.091	14665	999.35	1.354
0.15	12406	998.30	0.36	1.114	15509	999.12	1.349
0.1	12210	997.19	0.33	1.177	17054	998.77	1.360
0.07	12940	995.93	0.31	1.245	17159	997.88	1.417
0.05	14173	993.99	0.28	1.356	20119	997.28	1.422
0.01	22005	979.38	0.12	2.188	31243	987.73	1.814
0.009	23288	978.18	0.10	2.263	31345	986.90	1.856
0.007	26629	976.06	0.09	2.386	37305	986.55	1.818
0.005	30207	970.44	0.07	2.725	44290	983.58	1.950

**Table 3.** Results for 1-Ant optimizing the fitness function ONEMAX and random linear functions for different values of  $\rho$ . All numbers are the averages over 10 runs.

$\rho$	$T$	$\overline{mm}$	$\overline{P}$	$\overline{v}$	$f_{\max}^{(T)}$	$P^{(T)}$
1.0	34723	200.00	0.54	0.029		
0.9	35387	199.49	0.56	0.033		
0.8	31933	199.22	0.56	0.035		
0.7	34325	199.04	0.55	0.035		
0.6	31123	198.67	0.56	0.040		
0.5	34308	198.19	0.55	0.043		
0.4	34165	197.35	0.56	0.048		
0.3	33076	196.61	0.54	0.054		
0.2	32302	193.65	0.55	0.074		
0.1	32801	187.63	0.54	0.111		
0.095	33232	188.51	0.52	0.137		
0.09	32733	187.50	0.53	0.141		
0.085	30548	186.95	0.53	0.148		
0.08	33212	185.51	0.53	0.128		
0.075	32188	185.12	0.53	0.152		
0.07	31473	184.04	0.53	0.148		
0.065	29955	182.59	0.53	0.185		
0.06	32338	181.96	0.53	0.171		
0.055	32074	179.93	0.53	0.190		
0.05	32203	181.33	0.51	0.234		
0.045	33857	180.25	0.50	0.225		
0.04	32063	176.21	0.51	0.283		
0.035	30913	178.57	0.48	0.299		
0.03	30625	173.51	0.49	0.441		
0.025	27078	170.75	0.48	0.587		
0.02	32106	165.97	0.47	0.562		
0.015	31658	164.87	0.44	0.918		
0.01	32057	158.36	0.41	1.210		
0.009	31670	156.45	0.41	1.495		
0.008	30905	155.13	0.39	1.651		
0.005	34899	151.73	0.33	2.289		
0.0025	42780	141.70	0.25	4.988		
0.001	65270	137.68	0.15	12.389		

**Table 4.** Results for the MMAS optimizing the fitness function LEADINGONES for different values of  $\rho$ . All numbers are the averages over 10 runs.