A language over a one symbol alphabet requiring only O(log log n) space

Helmut Alt & Kurt Mehlhorn
Fachbereich 10
Universität des Saarlandes
66 Saarbrücken, West Germany

It is well known that the minimal growth function for the tape complexity of Turing machines is log log n [2]. In the literature, one can find essentially one example of a language requiring only O(log log n) space, namely

$L_0 = \{ \# bin(1) \# \; bin(2) \# \; bin(3) \# \dots \# \, bin(n) \# ; \; n \in \mathbb{N} \}$

where bin(i) is the binary representation of the integer i. In this note we describe a language over a <u>one</u> symbol alphabet having space complexity O(log log n).

Let $L_1 = \{ a^n;$ the smallest number q which does not divide n is a power of two$\}$

For every natural number n let q(n) be the smallest number which does not divide n.

<u>Lemma:</u> $\exists c > 0: \quad q(n) \le c \log n$

<u>Proof:</u> Let n be any natural number and let m = q(n). Then all primes less than m divide n and hence their product P divides them. By the prime number theorem of Gauss there are about $^{m}/_{\log m}$ primes less than m and hence P is larger than

$(^m/_{\log m})! \geq 2^{m/4}$ for sufficiently large m.

q.e.d.

As an immediate consequence of lemma 1 we obtain that the following recognition procedure has space complexity O(log log n).

<u>input</u> $n \in \mathbb{N}$;

$q \leftarrow 1$;

<u>repeat</u> $q \leftarrow q + 1$; $p \leftarrow n \bmod q$ <u>until</u> $p \neq 0$;

<u>if</u> q is a power of two <u>then</u> accept <u>else</u> reject;

It remains to be shown that L_1 is not regular. For $k \in \mathbb{N}$ let m_k be the least common multiple of the numbers $1,\ldots,2^k-1$. Then $a^{m_k} \in L_1$ for all k. Furthermore, since for infinitely many k there exists a prime between 2^k and 2^{k+1}, $a^{2m_k} \notin L_1$ for infinitely many k. Assume now that L is regular. Then $L_1 = \bigcup_{i \in J} s_i + t_i \mathbb{N}$ for some finite set J and some $s_i, t_i \in \mathbb{N}_0$. For sufficiently large k every non-zero t_i divides m_k and hence with $m_k \in s_i + n_i \mathbb{N}$ also $2m_k \in s_i + t_i \mathbb{N}$. Contradiction!

<u>Conclusion:</u> We have exhibited a non-regular language over a one symbol alphabet requiring only O(log log n) space to recognize. This example was found in the futile attempt to extend the result from [1]. There we showed:

Theorem: Let L be context-free and let u,v,w,x,y be words with $L \cap uv^*wx^*y$ non-regular. Then log n is a lower bound for the space complexity of L.

[1] H. Alt & K. Mehlhorn: Untere Schranken für den Platzbedarf bei der kontext-freien Analyse, Techn. Bericht, Fachbereich 10 der Universität des Saarlandes, 1975

[2] J. Hopcroft & J. Ullman: Formal Languages and their Relation to Automata, Addison-Wesley, Reading, Mass., 1969

****

How to Gracefully Number Certain Symmetric Trees

Harold N. Gabow
Department of Computer Science
University of Colorado
Boulder, Colorado 80309

Let G be an (undirected) graph, with vertex set V and edge set E. A vertex-numbering assigns a distinct non-negative integer to each vertex of G. We regard a vertex and its vertex number as identical objects; thus V is the set of all vertex numbers. With an edge (v,w) we associate the edge number $|v-w|$. We identify E with the set of all edge numbers. A graceful numbering of a tree with n vertices has $V = \{i|0 \le i < n\}$, $E = \{i|1 \le i < n\}$. For example, the trees in Figure 1(a) and 1(c) are gracefully numbered.

In [C,R], constructions are given for gracefully numbering a restricted class of symmetric trees. We give a more general construction, that gracefully numbers a tree if the subtrees of the root are all isomorphic to one gracefully numbered tree. We indicate other trees that can be gracefully numbered by our technique.