

Perceptual Evaluation of Tone
Mapping Operators with Regard to
Similarity and Preference

Frederic Drago, William L. Martens,
Karol Myszkowski, and Hans-Peter Seidel

MPI-I-2002-4-002

October 2002

FORSCHUNGSBERICHT RESEARCH REPORT

MAX-PLANCK-INSTITUT
FÜR
INFORMATIK

Stuhlsatzenhausweg 85 66123 Saarbrücken Germany

Author's Address

Frederic Drago, Karol Myszkowski, and Hans-Peter Seidel
Max-Planck-Institut für Informatik
Stuhlsatzenhausweg 85
66123 Saarbrücken
Germany
`{drago,myszkowski,hpseidel}@mpi-sb.mpg.de`

William L. Martens
University of Aizu
Tsuruga Ikki-machi
Aizuwakamatsu City
965-8580 Fukushima, Japan
`wlm@u-aizu.ac.jp`

Abstract

Seven tone mapping methods currently available to display high dynamic range images were submitted to perceptual evaluation in order to find the attributes most predictive of the success of a robust all-around tone mapping algorithm. The two most salient *Stimulus Space* dimensions underlying the perception of a set of images produced by six of the tone mappings were revealed using **I**ndividual **D**ifferences **S**CALing (INDSCAL) analysis; and an ideal preference point within the INDSCAL-derived *Stimulus Space* was determined for a group of 11 observers using **P**REference **M**APping (PREFMAP) analysis. Interpretation of the INDSCAL results was aided by pairwise comparisons of images that led to an ordering of the images according to which were more or less natural looking.

Keywords

Tone Mapping Operators, Visual Perception, High Dynamic Range Imaging, Global Illumination, Preference Mapping

1 Introduction

For years, 24-bit images effectively using 256 values for each color channel were considered accurate enough to be used in the creation and display of photorealistic imagery. On the other end, photographers knowing the limitation of cameras and films would never attempt to photograph a scene including a light source placed in the camera's field of vision. However, as a result of global illumination rendering and high dynamic range (HDR) imaging [DM97], it is now common to generate images spanning a huge range of luminance. Dealing with such images in electronic form requires extending file formats that are restricted to too few luminance values and colors. An even more serious issue is effective displaying/printing of those images using media with limited dynamic range (both in reproducible luminance values and color gamut).

The first limitation was successfully addressed in different pixel encoding formats. A simple approach relies on using three floating point numbers for each pixel RGB values, however, this leads to excessive file sizes. The HDR image size can be reduced to four bytes for each pixel using the RGBE format [War91], in which a common exponent and a mantissa are assumed for each channel thus allowing a wide dynamic range with little storage overhead. Another format, the *logLuv* encoding for *tiff* images [Lar98] separates a logarithmic representation of luminance and a *CIE*(u, v) representation of color. In the 32-bit version of *logLuv*, luminance information is encoded using 16 bits, covering the full range of perceivable luminance. The second, most restrictive, limitation, is that current CRT displays span a very short dynamic range – often less than two orders of magnitude. Printers are even more limited, and while film-based media offer better luminance mapping capabilities, they still span only a portion of the visual range.

This paper examines seven tone mapping techniques, which offer different approaches to address these limitations. The problem of real world luminance mapping was earlier encountered in lighting engineering when the results of lighting simulation obtained in photometric units had to be displayed on a CRT device in the form of scene images [MNM84]. This problem was identified in the computer graphics field by the pioneering work of Tumblin and Rushmeier [TR93].

The basic idea is to map the luminance range of an image along with its displaying method while keeping consistent scene characteristics. Ideally, the human impressions evoked while observing the real world scene and the corresponding displayed image should be the same.

In the last decade a number of tone mapping algorithms have been proposed and others are expected in the near future. However, a sound evaluation methodology based upon human perception of tone mapping results has not been well established. At this stage in research and development, an exploratory rather than confirmatory approach is needed. This is because, in the study of complex, naturalistic stimuli, it is often the case that an explicit psychophysical theory predicting relationships between such visual stimuli and perceptual responses is not available, or only prematurely specified. Under these circumstances it can be beneficial to collect subjective data about how perceptually similar or dissimilar stimuli are without specifying the ways in which stimuli may differ from one another. In the presence of data on such global differences between stimuli, the use of exploratory techniques, such as **INDividual Differences SCALing** (IND-SCAL), can be quite valuable. It is instructive to read an early explanation of the role of multidimensional perceptual scaling in this context from Torgerson's book [Tor58]:

“The traditional methods of psychophysical scaling presuppose knowledge of the dimensions of the area being investigated. The methods require judgments along a particular defined dimension . . . In many stimulus domains, however, the dimensions themselves, or even the number of relevant dimensions, are not known. What might appear intuitively to be a single dimension may in fact be a complex of several . . . Other dimensions of importance may be completely overlooked. In such areas the traditional approach is inadequate . . . This model differs from the traditional scaling methods in two important respects. First, it does not require judgments along a given dimension, but utilizes, instead, judgments of similarity between the stimuli. Second, the dimensionality, as well as the scale values, of the stimuli is determined from the data themselves.”

An example of successful computer-graphics-related application of **MultiDimensional Scaling** (MDS) analysis was presented by Pellacini et al. [PFG00], who used this method to uncover the most perceptually salient parameters for controlling a light reflection model. Similarly, the goal of the current project is the exploration of human response to a set of tone mapping operators with particular emphasis on their performance in producing natural looking and appealing images. Seven algorithms available at present were evaluated. It is hoped that

the methodology employed here can be used effectively to evaluate subsequently developed tone mappers as well.

The remainder of this paper is divided into three parts; Section 2, describes the tone mapping algorithms we tested during our research. Section 3, is dedicated to describing the procedure for experimental data collection, and the results of experimental data analysis are presented in Section 4.

2 Tone Mapping Operators

Here we briefly characterize the seven different tone mapping techniques, chosen for our experiments. A more complete survey of existing tone mapping techniques can be found in [MNP97, Tum99, McN01]. In our experiments we focused on the problem of compressing the luminance range in order to match it with the limited range of the display device. Effectively, we investigated the abilities of the tone mapping algorithms to adjust brightness and contrast of the processed scenes in order to preserve their original appearance on the display device. We did not take into account other characteristics of the human visual system (HVS) such as time adaptation to darkness, change in visual acuity, and effects like veiling glare, which are readily modeled by some tone mapping techniques that we considered.

For our experiments we chose well-established and commonly used classic tone mapping algorithms proposed in [TR93],[THG99], [Sch94] (we chose the Uniform Rational Quantization approach), [FPSG96] (which generalizes the approach proposed in [War94a]), and [LRP97]. Those algorithms are categorized as global methods because a tone mapping function is chosen once per processed scene based on its characteristics and is applied for all pixels in the output image. A number of techniques exploit the local adaptation of the human visual system, and the level of luminance range compression is decided locally based on the corresponding image characteristics. Such algorithms are called local, but unfortunately they often cause undesirable halo-like artifacts. For this reason we did not consider early local algorithms such as those presented in [CHS⁺93], [TO97], and [Sch94](Non-Uniform Rational Quantization). However, we included to our study carefully selected local algorithms such as Retinex introduced by Land and McCann [LM71], Low Curvature Image Simplifier (LCIS) [TT99], and Photographic Tone Reproduction (PTR) [RSSF02]. We excluded from our study the layering method [THG99] which is not suitable for the processing of HDR photographs because it requires detailed information concerning illumination and reflectance properties of scene surfaces. To produce consistent color output, for all tested algorithms we used the color handling approach proposed by Schlick [Sch94].

2.1 Revised Tumblin-Rushmeier Tone Reproduction Operator

The tone mapping algorithm proposed by Tumblin and Rushmeier [TR93] is rightly considered the reference in computer graphics. This operator introduced a model of brightness preservation based on a mathematical model of human vision by Stevens and Stevens [SS60]. The goal is to keep a constant relationship between the brightness of a scene perceived on a display and its real counterpart, for any lighting condition. We used a revision of the original method [THG99] which essentially eliminates the possible contrast anomalies in very dark scenes and improves the reduction of high contrast in extremely bright scenes. Tumblin applied this method to portions of the scene in his foveal method [THG99], we tested it on the whole scene like the other global tone mapping methods included in our survey. By using the logarithmic average of scene luminance as input parameter, we found the resulting images to give a good representation of the original scene in every tested case.

2.2 Uniform Rational Quantization

The original approach proposed by Schlick [Sch94], is a global tone mapping function aimed at rendering realistic looking images in every possible lighting conditions. It preserves contrast in dark regions and compresses high luminance values to avoid clipping. Only three parameters are required, the highest (for non primary light source) and lowest luminance values in the scene and a “just noticeable difference” factor, in fact the darkest displayable non black grey level visually selected by the user during screen calibration. For computer generated scenes, this tone mapping does not require any special input from the viewer, but in the case of scenes assembled from photographs, the highest luminance parameter can hardly be found. Schlick’s method works very well to show detailed features in high contrast, highly illuminated scenes.

2.3 Visual Adaptation Model

A psychophysical model of visual adaptation was proposed by Ferwerda et al. [FPSG96], this method takes into account the characteristics of the human mechanisms of vision. Changes in threshold sensitivity, visual acuity, temporal adaptation to light and darkness, and changes in color appearance were implemented in a coherent model. We tested only the luminance mapping part which is based on the contrast threshold function previously proposed by Ward [War94a]. For

photopic viewing conditions, a function is used to map luminance of the scene to the display. Dark scenes (scotopic vision) are mapped with another function using the light sensitivity characteristics of the rods. For mesopic conditions, both the photopic and scotopic functions are calculated and combined. We used the world adaptation luminance, based upon the log of the scene average luminance and a value of 25 cd/m^2 for the display adaptation luminance. In our particular case, images of fairly high contrast, brightness clipping was often a drawback of the method (this being due to a choice to bring out scene features in low luminance regions).

2.4 Histogram Adjustment

The goal of the histogram adjustment method [LRP97] is to produce visually accurate images based on human contrast sensitivity. By adaptively compressing contrast within a local adaptation model, the histogram adjustment method overcomes the short dynamic range of current displays. The complete method made available by Ward in the Radiance rendering system [War94b] includes human vision characteristics, but since our goal was to optimize visibility we used for this survey only the automatic mapping of scene luminance. Since we did not use the human contrast sensitivity function of the algorithm, a linear ceiling function is applied during the histogram equalization process. Regardless of the characteristics of the scene, all parts of the resulting image are visible, giving the greatest possible dynamic range. Figure 2.1 shows the histogram of two images from different scenes after automatic tone mapping by the histogram adjustment method.

2.5 Low Curvature Image Simplifier

Tumblin and Turk [TT99] presented the Low Curvature Image Simplifier (LCIS) method to optimize the display of high contrast scenes on low contrast media. This local adaptation multi-pass method reduces the scene features to smooth regions bounded by sharp gradient discontinuities, then scenes processed with different levels of details are added to form the resulting low contrast highly detailed image. Close to anisotropic diffusion, LCIS could be compared to the behavior of fluid flowing from the scene boundary regions, forming ridges on shock lines and flattening smoother regions. Although the goal of LCIS is not just tone mapping, but rather generation of low contrast high detail images, we considered LCIS in our experiments because of the attractive and unusual appearance of images processed using this method (Figure 2.2).

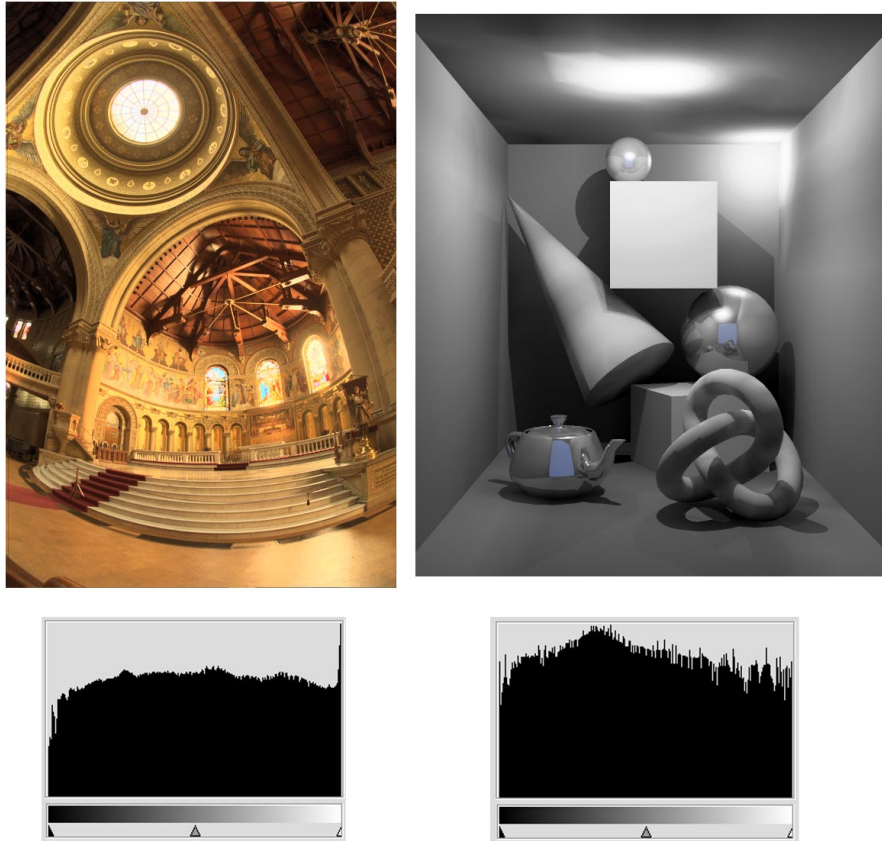


Figure 2.1: The Memorial church and the box scenes after histogram adjustment, all possible pixel intensity values are present in the images. Although, the two scenes are quite different, the histograms of the resulting images show many similarities.

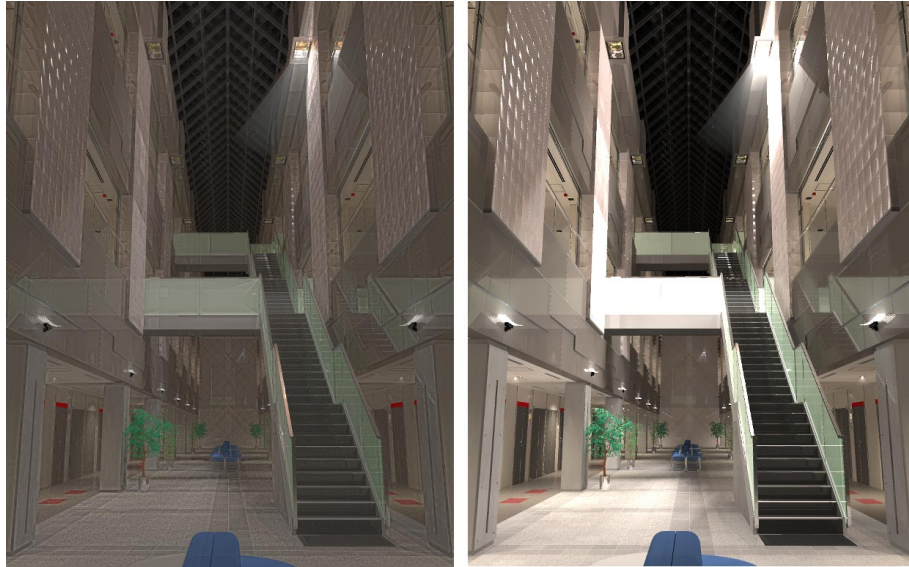


Figure 2.2: A highly detailed low contrast image result of the LCIS method (left), and the same scene computed by the Visual Adaptation Method resulting in a very high contrast image (right).

2.6 Retinex

The Retinex algorithm introduced by Land and McCann [LM71] aims at processing the human sensory response to lightness in a scene, it does not calculate physical or perceived reflectance. Interesting to us is the fact that Retinex naturally maximizes the range of luminance and improves visual differentiation at all ranges of radiance. The algorithm is not well known to the computer graphics community, but electronic imaging researchers are intensively working on improvements of the original idea, notably by using a multiscale approach [JRW97]. One of the goals of the performed research is to integrate Retinex in digital cameras for automatic correction and enhancement of badly exposed or white balanced images. One of Retinex particularity is that it addresses the color constancy problem. We did not take advantage of this feature since our goal was only to adjust the dynamic range of a scene for an optimal display.

We tested the Retinex algorithm proposed in [FM83], ported from the Matlab code [FCM00] to our framework. This scheme computes interactions between pixels, first for a long distance and then halving distances until the unit pixel size is reached. A ratio of the lightness of the two distant pixels is multiplied by the lightness result of the previous Retinex iteration at the current location. A reset value is applied if the lightness value is greater than a maximum allowed lumi-

nance. The intermediate result is averaged with the result of the previous iteration and will be used to calculate an average with the result of the next iteration. The original algorithm follows a square spiral path in a clockwise way, we also used a complementary counter clockwise path to balance the first following Coopers recommendation [Coo02]. This doubles calculation time, but seems to be effective for balancing dark halo artifacts and preserving many details. To further reduce black halos (Figure 2.3) common to local tone mapping operators [CHS⁺93], we had to scale down the high luminance values to the maximum displayable luminance of the output display. The number of Retinex iterations controls contrast and dynamic range compression of the resulting image. The parameters used are the number of iterations, the maximum display luminance (we set it at 100 cd/m^2 following the recommendations of the display manufacturer), and an exposure factor. 33 Retinex iterations were found to be optimal [FC01], so we used this number in our experiment.



Figure 2.3: Changes made to Retinex for mapping very high contrast scenes. Dark halos around very high luminance areas (left image) were corrected by scaling world luminance to the display range (right image).

2.7 Photographic Tone Reproduction

Reinhard et al. [RSSF02] proposed the Photographic Tone Reproduction method, which is based on photographic film development and print process. Its aim is to automate the proven techniques used for years by photographers to transfer images from film to a displayable medium. The luminance of the scene can be initially mapped with a global tone mapping function to bring every pixel value into a displayable range. Luminance is adaptively scaled, reducing high value while remaining low value somewhat constant. To further enhance the quality of the resulting image, a local adaptation operator can be used. A parallel is made with the photographic dodging-and-burning techniques allowing a different exposure for each part of the image being printed. To automate this process, low contrast regions are found by using a center-surround function operating at different scales and image coordinates. When an ideal average value of the area surrounding a pixel is found, the tone mapping function can be applied locally. The automatic dodging-and-burning method enhances contrast and details in the image while preserving the scene overall luminance characteristics (Figure 2.4).

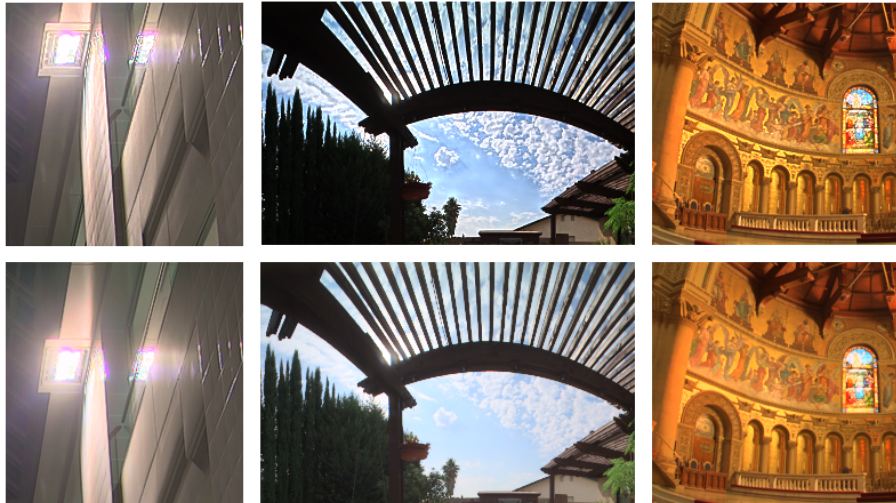


Figure 2.4: Comparison of local versus global applications of the Photographic Tone Reproduction operator, the top row of images tone mapped with the local model offer much more contrast and details.

3 Experimental Framework

The goal of the perceptual evaluation was to measure the supra-threshold differences among images processed with different tone mapping techniques for various scenes. The most important answer given by the subjects is a distance or difference value (on a slider scale ranging from 0 to 100) between two images shown on the CRT screen. Two additional questions were asked: “of the two images presented, which one seems to be more natural ?” and finally a personal preference choice between the two stimuli: “of the two images presented, which one is your preferred ?” We rated the visual perception of eleven subjects, all of them computer graphics proficient, but except for the authors without any clue about the goals of the experiment. It would have been interesting to test different groups of subjects such as for example photographers, broadcast professional or people without any connections to our research.

3.1 Selection of the Scenes

Four scenes all spanning very big ranges of luminance were selected (Figure 6.1), beside their dynamic range, they also presented very different characteristics in terms of environment. The first scene “ATRIUM” represents an atrium at the University of Aizu and is the result of a Bidirectional Monte Carlo path tracing rendering. Since the accuracy of the global illumination simulation was found satisfying in a previous study [DM01], we used luminance values measured by the rendering software for tone mapping. This environment was the most challenging in terms of lighting characteristics, with parts in very dark shadows opposed to a powerful luminaire casting light to the main architectural features. The second scene “MEMORIAL”, the Stanford Memorial church courtesy of Paul Debevec, is again an interior scene but naturally lit by the sun shining through stained glass windows. It spans a wide dynamic range and has many detailed features. Also being illustrated in many papers, we could use it to compare our implementations of tone mapping operators to others. The third scene “PANORAMA” is a panoramic

photograph of an outdoor environment, the high dynamic range photograph was taken by a Spheron [Sph] panoramic high dynamic range digital camera. In this scene the sun is partially hidden by the veranda, which makes its proper shooting virtually impossible using standard photographic equipment. Finally, we considered a totally different kind of scene, a simple computer generated environment “BOX” made of geometric models placed in a box. The lack of details and real-world connotations made this environment very difficult for perceptual rating.

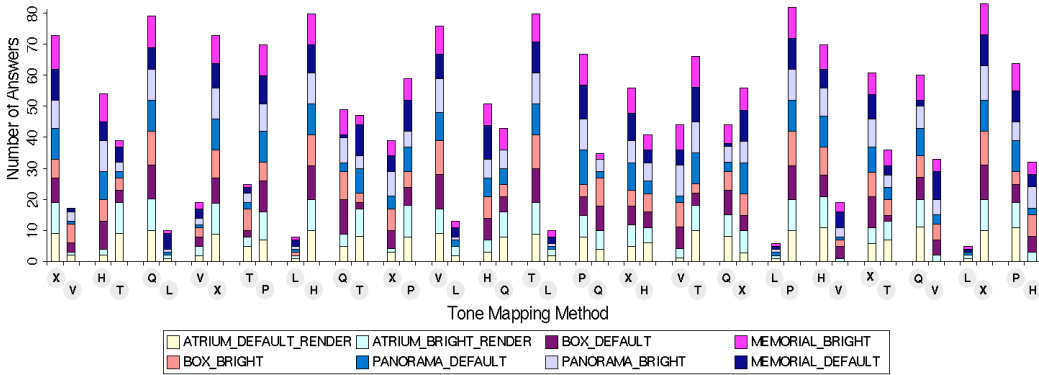


Figure 3.1: Detailed graph of answers to the question: “Of the two images presented on the CRT screen, which one seems to be more natural?”. The answers for the second question: “Of the two images presented on the CRT screen, which one is your preferred?” are not shown here since the total repartition of answers for each tone mapping method was only slightly different in both cases. The letter in each circular symbol identifies the tone dapper applied: **L**ow Curvature Image Simplifier (**L**), **R**evised **T**umblin and **R**ushmeier (**T**), **P**hotographic Tone **R**eproduction (**P**), **U**niform Rational **Q**uantization (**Q**), **H**istogram **A**djustment (**H**), **R**etine**X** (**X**), and **V**isual **A**daptation (**V**).

3.2 Experimentation Methods

For a most comprehensive evaluation, it was decided to conduct comparisons with two different sets of images. A first set consisted of scenes tone mapped using the algorithms with suggested default parameters if appropriate (we use the suffix “_DEFAULT” to refer to the corresponding images e.g., “BOX_DEFAULT”), and a second set for which we tried to adaptively match the brightness of each image to the others, by calculation of the average pixel value of the tone mapped image (we

mark those images with the suffix “_BRIGHT”). As a brightness value reference, we used the average of pixel values from the output of the histogram method. Brightness adjustments were done by modifying the log average luminance of the scene for the Revised Tumblin-Rushmeier and Visual Adaptation operators, the low luminance value for the Uniform Rational Quantization, and the exposure setting in LCIS, Retinex and the Photographic Tone Reproduction methods. Scenes were displayed in a random order, and the two sets were mixed. In summary we had four scenes mapped with seven different techniques and displayed twice. Since pairwise comparisons were made, the subjects had to respond to a total of 168 stimuli. For better consistency, an ICC custom profile of the CRT display was built and the experiment was done in a light controlled environment. The Web page with image pairs prepared for the perceptual evaluation can be found online at this address: <http://www.mpi-sb.mpg.de/resources/tmo>.

Two types of formal experimental trials were completed by all members of the group of subjects. First, observers made global dissimilarity ratings on a 100-point scale for all pairwise comparisons of the images. Second, they made a choice of which image in each pair dominated the other with respect to each of two aspects: the first being a choice of which of the two images looked most natural (Figure 3.1), the second being a choice of which of the two was most preferred in the most global sense, that is, without respect to any particular property. A straightforward comparison of the number of times each tone mapping operator was chosen as more preferred over others shows consistently higher rankings for the Photographic Tone Reproduction followed by the Histogram Adjustment method. The two techniques seem to offer greater flexibility in their application and managed to produce well balanced images accordingly to the viewer expectations. LCIS was not adapted to our experiment, where emphasize on naturalness of the image was made, its capabilities are unique but it seems that the lack of contrast of the images it generated was perceived as too unrealistic. The Uniform Rational Quantization proved to be well adapted for every kind of lighting condition using only its default parameters. Retinex is capable of producing even more detailed images, but due to our very high dynamic range images a tradeoff had to be made here in favor of the “black halo” removal. The revised Tumblin-Rushmeier operator was usually preferred to the Visual Adaptation model, the emphasize on contrast preservation of the latter proved to be a weakness in scenes with very high luminosity where features in areas close to light sources were washed out.

Before completing the data acquisition, we initiated a discussion with the participants, our main concern being to elicit a verbal description of the criteria used in their preference choices. As is described in the next section of this paper, dissimilarity ratings were collected in order to obtain a low-dimensional spatial configuration characterizing the stimuli, and the naturalness choice data were used to aid in the interpretation of the obtained configuration (along with two other at-

tribute ratings, contrast and level of detail). The preference choice data were used to identify an ideal preference point within the obtained configuration.¹

¹An in depth description of the multidimensional perceptual scaling analyses employed here, using PREFMAP and INDSCAL algorithms, is beyond the scope of this paper. The interested reader is directed to the excellent introduction found in [BG97]. For computational details, the reader should see [Car80]

4 Experimental Results

4.1 Individual Differences Scaling Analysis

Dissimilarity ratings obtained for all pairwise comparisons of images made by 11 observers were submitted to **INDividual Differences SCALing** (INDSCAL) analysis using the **ALSCAL** procedure of the SPSS software package [SPS99]. Separate INDSCAL analyses were performed for each of 24 images (six tone mappers by four scenes). Because the LCIS stimuli were always rated most dissimilar to all other stimuli, only data for six of the seven tone mappers was included in this analysis. By excluding the data collected regarding the LCIS stimuli, a better spatial configuration solution could be obtained characterizing the dissimilarities between the remaining six tone mappers. Figure 4.1 shows the two-dimensional (2D) “group” solution that resulted from this subsequent INDSCAL analysis. Coordinates on the two dimensions of this *Stimulus Space* configure the stimuli such that the Euclidean distances between the stimuli match well the obtained dissimilarity judgments. Note that *Stimulus Space* axes have been labeled based upon correlation of the dimensional coordinates with independently generated attribute ratings (as explained below).

4.2 Naturalness and Detail Vector Unfolding

The obtained pairwise naturalness choices for 11 observers were processed to generate an ordinal scaling of naturalness for each of the stimuli. In a subsequent session, choices from five additional observers were collected regarding which stimuli appeared to have higher contrast and higher detail. Again, an ordinal scaling for these two attributes was derived for all of the stimuli. These scale values were submitted to **PREFerence MAPPing** analysis using the **PREFMAP-3** software developed by J. Meulman, W. Heiser, and J. D. Carroll at Bell Laboratory [MHC86]. **PREFMAP** can relate any sort of dominance information to a provided spatial configuration of points, where the data can indicate any sort of condi-

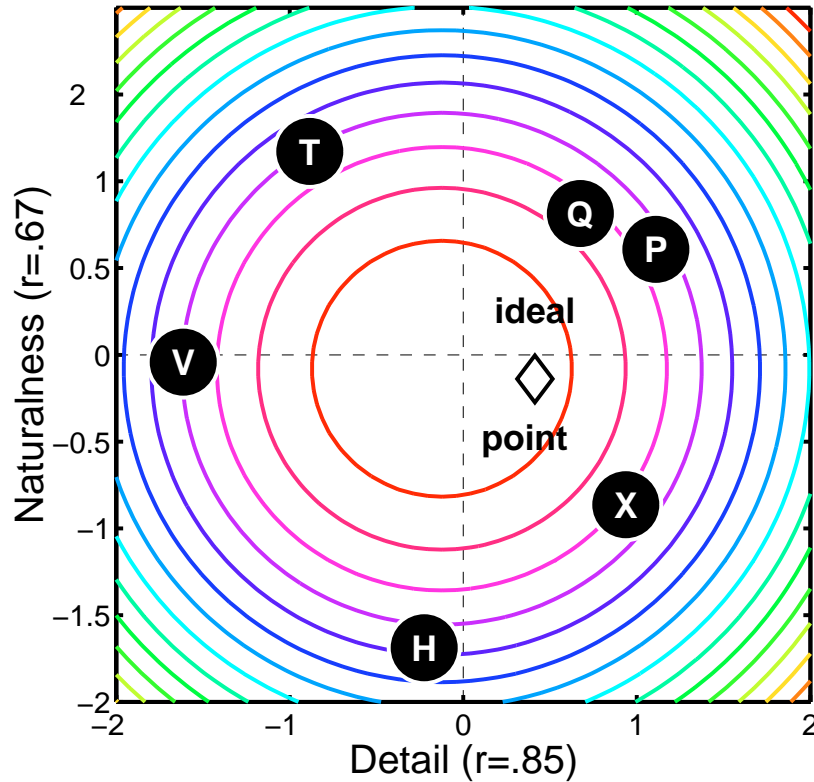


Figure 4.1: The 2D “group” *Stimulus Space* solution based upon INDSCAL analysis of dissimilarity ratings for six tone mappers applied to four scenes (24 images total). The letter in each circular symbol identifies the tone mapper applied: Revised **T**umblin and Rushmeier (**T**), **P**hotographic Tone Reproduction (**P**), Uniform Rational **Q**uantization (**Q**), **H**istogram Adjustment (**H**), Retine**X** (**X**), and **V**isual Adaptation (**V**). The concentric circles correspond to iso-contrast contours, and the black diamond symbols plots the “ideal” preference point that resulted from PREFMAP ideal-point unfolding. The detail ratings correlated most highly with dimension D1, and the naturalness ratings correlated most highly with dimension D2.

tional dominance relation among the compared stimuli. The PREFMAP analysis requires two sorts of input data, the first in the current case being the naturalness scores for each of the tone mappers, and the second being the *target configuration matrix*, which in the current study was provided by the INDSCAL-derived *Stimulus Space*. The correlation of the obtained scale values with coordinates of the INDSCAL-derived *Stimulus Space* dimensions D1 and D2 are indicated along with the axis labels in Figure 4.1. Detail scale values were most closely associated variation along dimension D1, and naturalness scale values were most closely associated variation along dimension D2. Contrast scale values were not linearly related to either of the dimensions: Note that each tone-mapper plotting symbol sits on a ring of constant apparent contrast (the plot includes iso-contrast contours, the smallest contour in the center of the plot indicating the lowest level of apparent contrast ratings). To understand how the coordinates of these symbols relate to a linear scale for apparent contrast rating, the reader is asked to imagine what would happen if the 2D plotting surface were to be grasped at the interior low-contrast point, and then pulled through a small hole. The symbols would now be arranged on the linear contrast scale that would fit very nearly the obtained ratings. The smooth surface was fit via non-linear regression of contrast on the INDSCAL-derived coordinates (and additional smoothing was required in deriving extrapolated values of apparent contrast). It is worth noting that a similar non-linear relation between image naturalness judgments and contrast judgments was obtained by [Jan01] (see Figure 2.6, p. 22). In his psychophysical evaluation of a set of images of natural scenes, both the perceived image quality and naturalness peaked midway between minimum and maximum apparent contrast.

These perceptual dimensions have an immediate interpretation in terms that are familiar to graphics researchers, and this interpretation also supports our more detailed results based upon analysis of human judgments in gathered in controlled experiments. Dimensions D1 and D2 combined with the contrast scale shown in Figure 4.1 separate the six tone mappers into three distinct groups (or regions). The first group consists of the Photographic Tone Reproduction (P), Uniform Scaling Quantization (Q), and Retinex (X) methods. Images processed using these methods feature well preserved high spatial frequency details and similar levels of the overall contrast reproduction. The Retinex (X) method performs worse than the other two methods in terms of naturalness of the image appearance. The second group contains the Visual Adjustment (V) and the Revised Tumblin-Rushmeier (T) methods, which lead to naturally looking images with high overall contrast. Such natural appearance is no doubt a consequence of the perceptual basis of these two tone mappers. However, these methods suppress more spatial details than the methods from the first group. The Histogram Adjustment (H) seems not to belong in a group with any of the other tone mapping techniques. This method produces images of very high contrast and preserves relatively well

spatial details, however, the resulting images appearance is considered as not very natural.

4.3 Preference Mapping

The obtained pairwise preference choices for 11 observers were first transformed into “dispreference scores” for each of the tone mappers. The derived dispreference scores were then submitted to **PREF**erence **MAP**ping analysis, but in contrast to the vector unfolding model used for the naturalness values, the ideal point model was employed here. This **PREFMAP** analysis is different from a straight-forward correlational approach in that it is capable of modeling *single-peaked* response functions. This means that a single ideal preference point can be found in the space populated by a small number of stimuli for which preference dominance information is available. Though the *Stimulus Space* is only sparsely populated, the ideal point plotted in Figure 4.1 shows where that ideal point is located relative to the stimuli. Of course, the exact coordinate values will change as other stimuli are added into the analysis, but the relative positions of the stimuli should remain stable. Furthermore, the orientation of the axes relative to the stimuli should also remain stable within a reasonable margin of error due to individual differences between observers, as explained in the next section. To understand how the coordinates of these points relate to the ideal preference point, the reader is again asked to imagine what would happen if the 2D plotting surface were to be grasped at an interior point, but this time it would be the ideal point. Pulling the surface through this small hole would line the six tone mappers up on an ordinal scale for image preference, with Photographic Tone Reproduction (P), Uniform Scaling Quantization (Q), and Retinex (X) located nearest to maximum preference.

4.4 Subject Space

The **INDSCAL** analysis performed here operated upon multiple sets of dissimilarity data collected from a number of observers (subjects). While sets of similarity data can be averaged across subjects to obtain one aggregated dataset for submission to classical **MultiDimensional Scaling** (**MDS**) analysis [TM63], this paper teaches the advantages provided by **INDSCAL** for the multiple sets of similarity data, without requiring the assumption of a homogeneous group of subjects, sharing a single common perceptual structure for the stimuli. The two primary advantages of **INDSCAL** are as follows:

- 1 **INDSCAL** provides a quantitative characterization of the individual differences that exist within a group of experimental subjects, based upon dissimilarity

judgments obtained from each subject.

- 2 INDSCAL provides an inherently unique configuration that requires no further analysis to find a meaningful rotation, in contrast to the orientational ambiguity inherent to classical MDS and factor analysis.

Interpreting the results of classical MDS is problematic because the solution can be rotated through an arbitrary angle without violating the structure of the solution. Of course, inter-stimulus distances remain invariant under rotation of both classical MDS and INDSCAL solutions alike; but the orientation of the INDSCAL solution is determined by modeling agreement between subjects. INDSCAL is designed to separate those factors that are common to a group of subjects from the ways in which subjects differ, those individual differences being captured in terms of the differing emphasis (or weights) that each subject places on each of those common factors.

Whereas Figure 4.1 showed the “group” *Stimulus Space* solution revealing the perceptual dimensions that are common to all subjects, Figure 4.2 shows the INDSCAL-determined weights that each individual subject placed on the underlying dimensions in producing their inter-stimulus dissimilarity judgments. These weights define a *Subject Space* within which differences can be readily observed between individuals in their response to one set of stimuli, and can also be observed within individuals in their responses between different sets of stimuli. In Figure 4.2, the weights on *Stimulus Space* dimensions D1 and D2 are shown for four different scenes and for eleven different subjects. The weights for the “BOX_BRIGHT” scene were plotted using green circles as plotting symbols. It is evident that observers put relatively higher weights on dimension D2 for this scene in comparison to the three other scenes; indeed, this is the only abstract scene presented to the subjects.

However, the wide spread of weights in Figure 4.2 are not a problem for the analysis. Indeed, a result in which all subjects placed the same weights on each dimension would create a problem for finding a unique orientation for the axes of the *Stimulus Space*. Though weights depend both upon which stimulus set is presented and upon which individual subject is examined, the dimensions themselves are robustly determined.

4.5 Discussion

In a related study by Pellacini et al. [PFG00], the dimensionality of gloss perception was placed on a two-dimensional space having contrast gloss and distinctness-of-image for axis. Closer to our study, Rogowitz et al. [RFS⁺98] presented a

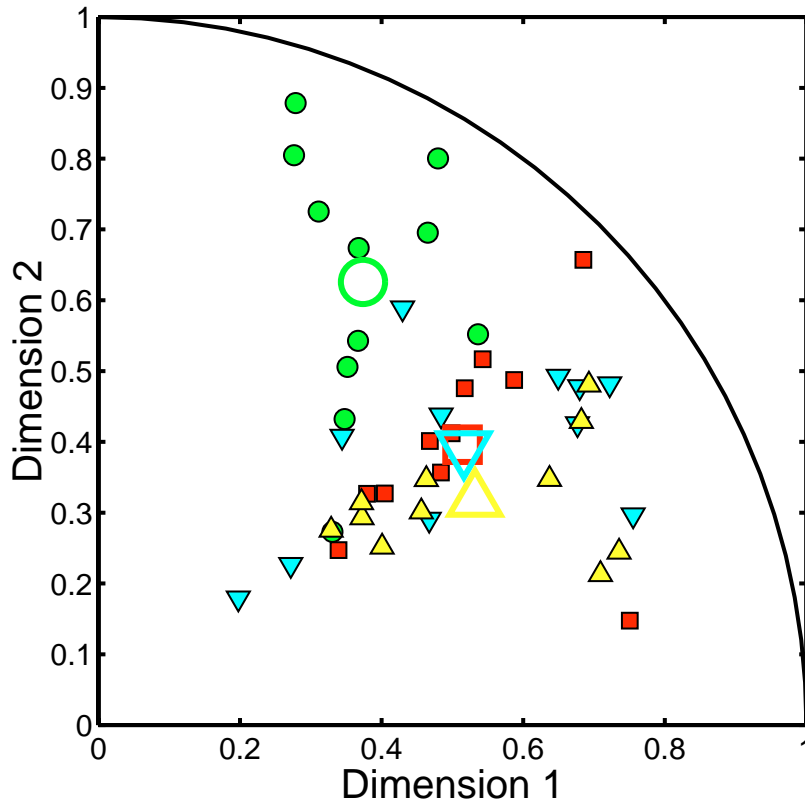


Figure 4.2: The *Subject Space* weights determined by INDSCAL analysis of dissimilarity ratings for six tone mappers applied to four scenes. The four sets of plotting symbols correspond to weights determined for each of the four scenes. For a given scene, smaller, color-filled symbols correspond to weights obtained for each of eleven subjects, and the larger open symbols correspond to the average subject's weight for each scene. Green circles are used for the "BOX_BRIGHT" scene, red squares for the "ATRIUM_BRIGHT" scene, yellow triangles for the "MEMORIAL_BRIGHT" scene and cyan-colored inverted triangles for the "PANORAMA_BRIGHT" scene. The unit-radius arc shows the limit for weight coordinates, since the weights cannot sum to a value greater than one. Symbols closer to the origin indicate that subjects put relatively lower weights on both of the dimensions of the "group" *Stimulus Space* solution.

perceptual evaluation of image similarity where 97 generic images from a photographic database were submitted to a psychophysical evaluation to study how human observers judge image similarity. They found that many dimensions could be used to classify images similarity, overall color appearance is one of the most characteristic along with semantic information. Other grouping such as natural opposed to man-made and less human-like versus more human-like were also proposed. Such a study is very useful to facilitate image classification and retrieval where automatic methods of indexing and searching large collections of digital images are actively researched. Since we made pairwise comparisons of the same scene, taking great care to not change color in the resulting images (indeed, comparison might have been easier using gray-scale images), and further eliminating brightness differences, the number of salient dimensions in our stimuli were reduced to just a very few. In a pilot study, we did not control for brightness differences so carefully, and consequently, the brightness differences dominated some of the more subtle perceptual attributes in which we were more interested. Consultation with the experimental subjects proved very useful here, as it turned out that two criteria were usually used to evaluate dissimilarity and naturalness could be identified as the “contrast” and “level-of-detail” attributes (for lack of better terms). Very low or high contrast images were usually rejected, totally black shadowed areas and scene features cropped to white were not well accepted, and the opposite was also true; that is, the vision of the luminaires geometrical features or details in shadowed areas, was not considered very realistic. Level-of-detail was a second more refined judgment, scene features such as visibility of clouds in the panorama or the stained glass and murals of the Memorial church were often criteria for deciding between two images. We propose here these two dimensions resulting from our research, they are not definitive choices but seems to match observers feeling and the repartition of tone mapping operators in the stimulus space we uncovered.

5 Conclusions

This paper presented the results of a perceptual evaluation of six commonly used tone mapping techniques. Multidimensional data analysis performed using the `INDSCAL` and `PREFMAP` methods revealed that the Photographic Tone Reproduction recently proposed by Reinhard et al. [RSSF02] produced images closest to the “ideal” preference point averaged across the subjects, at least within the small group of representative scenes tested. For these scenes, the results further showed that the Photographic Tone Reproduction, Uniform Rational Quantization, and Retinex methods produce better looking images in terms of subject preferences than the Visual Adaptation, Revised Tumblin and Rushmeier, and Histogram Adjustment methods.

The imagery resulting from two local methods Photographic Tone Reproduction and Retinex, as well as the global method Uniform Rational Quantization (featuring strong non-linear luminance compression), have been found to belong to the same group of methods, probably because these operators share many common characteristics in terms of contrast compression and detail reproduction which contribute significantly to the final image appearance. The global methods Visual Adaptation and Revised Tumblin and Rushmeier which use linear luminance scaling have been qualified as belonging to a different group of methods, even though their approach to tone mapping and the image they produce are very different. The Histogram Adjustment does not belong well to either of those two groups, but the resulting image appearance is closer to the former group, this is not surprising since this method emphasizes on displaying scene features at every luminance level and constantly offers the greatest tonal range. An interesting aspect we did not cover in this research is the comparison of a displayed image with the actual environment on site, since both the Visual Adaptation and Revised Tumblin and Rushmeier methods are based on the human visual system, a direct comparison might have significantly changed the perceived impressions.

These results are probably representative of typical applications in computer graphics and HDR photography, since this study employed various types of scenes (indoor/outdoor, daylight/artificial light, synthetic/photographs, simple geome-

try/complex natural environment scenes); nonetheless, further work is needed to determine the best interpretation of the *Stimulus Space* dimensions and the psychophysical relations that will help predict the success of tone mapping operators.

6 Acknowledgments

We would like to thank all the subjects who took part in our experiment, and Michael Goesele for calibrating the CRT display used in the experiment. We thank Scott Daly, Konstantin Kolchin, Przemek Rokita, and Annette Scheel for helpful discussions on tone mapping and image processing. Jack Tumblin, offered many helpful comments on our implementation of the LCIS method, Erik Reinhard gave us the opportunity to test the Photographic Tone Reproduction algorithm, and Brian Funt offered insightful comments about Retinex. The Stanford Memorial image was made available by Paul Debevec and the outdoor panorama image was provided by the SpheronVR company [Sph]. This work was supported in part by the European Community within the scope of the RealReflect project IST-2001-34744 “Realtime visualization of complex reflectance behavior in virtual prototyping”.

Bibliography

- [BG97] I. Borg and P. Groenen. *Modern Multidimensional Scaling: Theory and Applications*. Springer-Verlag, New York, 1997.
- [Car80] J. D. Carroll. Models and methods for multidimensional analysis of preference choice or other dominance data. In E. D. Lantermann and H. Feger, editors, *Similarity and Choice*, pages 105–155. Huber, Bern, Switzerland, 1980.
- [CHS⁺93] K. Chiu, M. Herf, P. Shirley, S. Swamy, C. Wang, and K. Zimmerman. Spatially nonuniform scaling functions for high contrast images. In *Graphics Interface '93*, pages 245–253, Toronto, Ontario, Canada, May 1993. Canadian Information Processing Society.
- [Coo02] Ted Cooper. Modifications to retinex to relax reset nonlinearity and implement segmentation constraints. In *Proc. IS&T/SPIE Symposium on Electronic Imaging: Science and Technology*, San José January 2002.
- [DM97] Paul E. Debevec and Jitendra Malik. Recovering high dynamic range radiance maps from photographs. In Turner Whitted, editor, *SIGGRAPH 97 Conference Proceedings*, Annual Conference Series, pages 369–378. ACM SIGGRAPH, Addison Wesley, August 1997. ISBN 0-89791-896-7.
- [DM01] Frédéric Drago and Karol Myszkowski. Validation proposal for global illumination and rendering techniques. *Computers & Graphics*, 25(3):511–518, June 2001. ISSN 0097-8493.
- [FC01] Brian Funt and Florian Ciuera. Control parameters for retinex. In *AIC 2001, Proc 9th Congress of the International Color Association*, June 2001.

- [FCM00] Brian Funt, Florian Ciuera, and John McCann. Retinex in matlab. In *Proc. IS&T/SID Eighth Color Imaging Conference*, pages 112–121, Scottsdale 2000.
- [FM83] Johnathan Frankle and John McCann. Method and apparatus for lightness imaging. *US Patent No: 4,384,336*, May 17 1983.
- [FPSG96] James A. Ferwerda, Sumant Pattanaik, Peter Shirley, and Donald P. Greenberg. A model of visual adaptation for realistic image synthesis. In Holly Rushmeier, editor, *SIGGRAPH 96 Conference Proceedings*, Annual Conference Series, pages 249–258. ACM SIGGRAPH, Addison Wesley, August 1996.
- [Jan01] Ruud Janssen. *Computational Image Quality*. Spie Press, Bellingham, Washington USA, 2001.
- [JRW97] Daniel J. Jobson, Ziaur Rahman, and Glenn A. Woodell. A multi-scale retinex for bridging the gap between color images and the human observation of scenes. *IEEE Transactions on Image Processing: Special Issue on Color Processing*, 6(7):965–976, July 1997.
- [Lar98] Gregory Ward Larson. Logluv encoding for full-gamut, high-dynamic range images. *Journal of Graphics Tools*, 3(1), pages 815–30, 1998.
- [LM71] Edwin Land and John McCann. Lightness and retinex theory. *Journal of the Optical Society of America*, 61(1):1–11, January 1971.
- [LRP97] Gregory Ward Larson, Holly Rushmeier, and Christine Piatko. A Visibility Matching Tone Reproduction Operator for High Dynamic Range Scenes. *IEEE Transactions on Visualization and Computer Graphics*, 3(4):291–306, 1997.
- [McN01] Ann McNamara. Visual perception in realistic image synthesis. *Computer Graphics Forum*, 20(4):211–224, 2001.
- [MHC86] J. Meulman, W. J. Heiser, and J. D. Carroll. PREFMAP-3 user’s guide. Technical report, Bell Telephone Laboratories, Murray Hill, NJ, USA, 1986.
- [MNM84] N.J. Miller, P.Y. Ngai, and D.D. Miller. The application of computer graphics in lighting design. *Journal of the Illuminating Engineering Society*, 14(1):6–26, 1984.

- [MNP97] Kresimir Matkovic, László Neumann, and Werner Purgathofer. A survey of tone mapping techniques. In *13th Spring Conference on Computer Graphics*, pages 163–170. Comenius University, Bratislava, Slovakia, June 1997.
- [PFG00] Fabio Pellacini, James A. Ferwerda, and Donald P. Greenberg. Toward a psychophysically-based light reflection model for image synthesis. In *Proceedings of ACM SIGGRAPH 2000*, Computer Graphics Proceedings, Annual Conference Series, pages 55–64. ACM Press / ACM SIGGRAPH / Addison Wesley Longman, 2000.
- [RFS⁺98] Bernice E. Rogowitz, Thomas Frese, John Smith, Charles A. Bouman, and Edward Kalin. Perceptual image similarity experiments. In *IS&T/SPIE Conf on Human Vision and Electronic Imaging III*, Proceeding of SPIE, volume 3299, pages 576–590, 1998.
- [RSSF02] Erik Reinhard, Michael Stark, Peter Shirley, and Jim Ferwerda. Photographic tone reproduction for digital images. In *SIGGRAPH 2002 Conference Proceedings*. ACM SIGGRAPH, Addison Wesley, August 2002.
- [Sch94] Christophe Schlick. Quantization techniques for the visualization of high dynamic range pictures. In Peter Shirley Georgios Sakas and Stefan Müller, editors, *Photorealistic Rendering Techniques*, Eurographics, pages 7–20. Springer-Verlag Berlin Heidelberg New York, 1994.
- [Sph] SpheronVR. <http://www.spheron.com/>.
- [SPS99] SPSS Inc., Chicago, WA. *SPSS Base 10.0 User's Guide*, 1999.
- [SS60] S.S. Stevens and J.C. Stevens. Brightness function: parametric effects of adaptation and contrast. *Journal of the Optical Society of America*, 50(11):1139A, November 1960.
- [THG99] Jack Tumblin, Jessica K. Hodgins, and Brian K. Guenter. Two methods for display of high contrast images. *ACM Transactions on Graphics*, 18(1):56–94, January 1999. ISSN 0730-0301.
- [TM63] L.R. Tucker and S.J. Messick. An individual differences model for multidimensional scaling. *Psychometrika*, 38:333–368, 1963.
- [TO97] T. Tanaka and N. Ohnishi. Painting-like image emphasis based on human vision systems. *Computer Graphics Forum*, 16(3):253–260, August 1997. ISSN 1067-7055.

- [Tor58] W.S. Torgerson. *Theory and methods of scaling*. Wiley, New York, 1958.
- [TR93] Jack Tumblin and Holly E. Rushmeier. Tone reproduction for realistic images. *IEEE Computer Graphics and Applications*, 13(6):42–48, November 1993.
- [TT99] Jack Tumblin and Greg Turk. LCIS: A boundary hierarchy for detail-preserving contrast reduction. In Alyn Rockwood, editor, *Siggraph 1999, Computer Graphics Proceedings, Annual Conference Series*, pages 83–90, Los Angeles, 1999. Addison Wesley Longman.
- [Tum99] J.E. Tumblin. *Three Methods For Detail-Preserving Contrast Reduction For Displayed Images*. PhD thesis, Georgia Institute of Technology, 1999.
- [War91] Greg Ward. Real pixels. *Graphics Gems II*, pages 80–83, 1991.
- [War94a] Greg Ward. A contrast-based scalefactor for luminance display. *Graphics Gems IV*, pages 415–421, 1994.
- [War94b] Gregory J. Ward. The radiance lighting simulation and rendering system. In *Proceedings of SIGGRAPH 94, Computer Graphics Proceedings, Annual Conference Series*, pages 459–472, Orlando, Florida, July 1994. ACM SIGGRAPH / ACM Press. ISBN 0-89791-667-0.

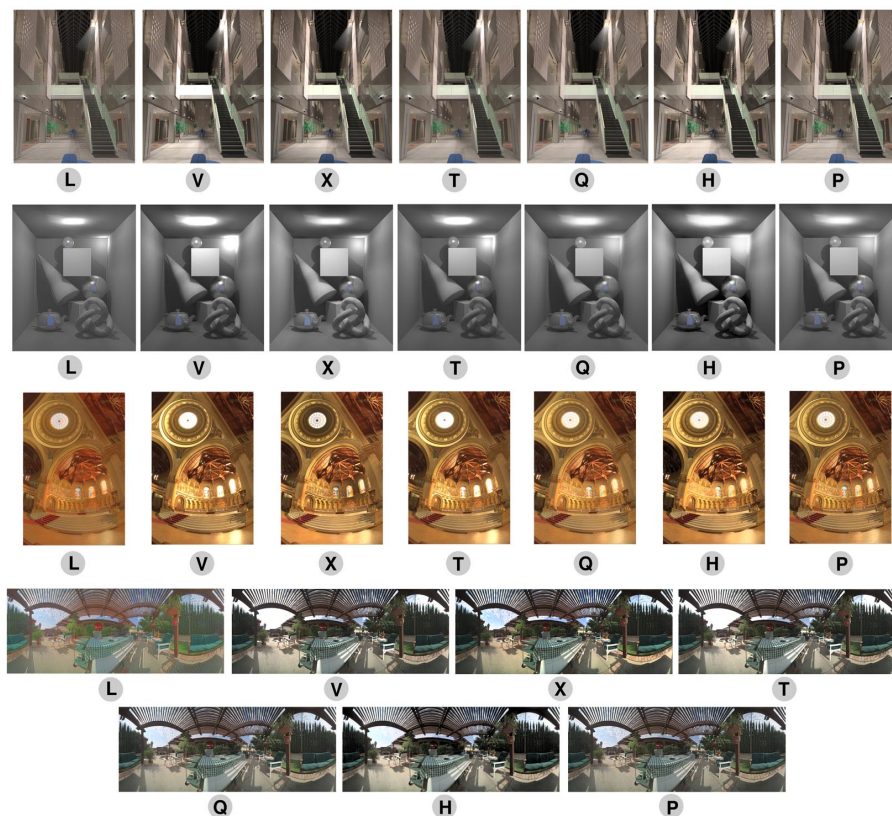


Figure 6.1: Global comparison of the seven tone mapping techniques used in the experiment. The tone mapping treatments for each scene was adapted to present an equal average image brightness. The letter in each circular symbol identifies the tone mapper applied: **L** Low Curvature Image Simplifier, **V** Visual Adaptation, **X** RetineX, **T** Revised Tumblin and Rushmeier, **Q** Uniform Rational Quantization, **H** Histogram Adjustment, and **P** Photographic Tone Reproduction.

Below you find a list of the most recent technical reports of the Max-Planck-Institut für Informatik. They are available by anonymous ftp from [ftp.mpi-sb.mpg.de](ftp://ftp.mpi-sb.mpg.de) under the directory `pub/papers/reports`. Most of the reports are also accessible via WWW using the URL <http://www.mpi-sb.mpg.de>. If you have any questions concerning ftp or WWW access, please contact reports@mpi-sb.mpg.de. Paper copies (which are not necessarily free of charge) can be ordered either by regular mail or by e-mail at the address below.

Max-Planck-Institut für Informatik
 Library
 attn. Anja Becker
 Stuhlsatzenhausweg 85
 66123 Saarbrücken
 GERMANY
 e-mail: library@mpi-sb.mpg.de

MPI-I-2002-4-002	F. Drago, W. Martens, K. Myszkowski, H. Seidel	Perceptual Evaluation of Tone Mapping Operators with Regard to Similarity and Preference
MPI-I-2002-4-001	M. Goesele, J. Kautz, J. Lang, H.P.A. Lensch, H. Seidel	Tutorial Notes ACM SM 02 A Framework for the Acquisition, Processing and Interactive Display of High Quality 3D Models
MPI-I-2002-2-008	W. Charatonik, J. Talbot	Atomic Set Constraints with Projection
MPI-I-2002-2-007	W. Charatonik, H. Ganzinger	Symposium on the Effectiveness of Logic in Computer Science in Honour of Moshe Vardi
MPI-I-2002-1-008	P. Sanders, J.L. Träff	The Factor Algorithm for All-to-all Communication on Clusters of SMP Nodes
MPI-I-2002-1-003	I. Katriel, P. Sanders, J.L. Träff	A Practical Minimum Scanning Tree Algorithm Using the Cycle Property
MPI-I-2002-1-002	F. Grandoni	Incrementally maintaining the number of 1-cliques
MPI-I-2002-1-001	T. Polzin, S. Vahdati	Using (sub)graphs of small width for solving the Steiner problem
MPI-I-2001-4-005	H.P.A. Lensch, M. Goesele, H. Seidel	A Framework for the Acquisition, Processing and Interactive Display of High Quality 3D Models
MPI-I-2001-4-004	S.W. Choi, H. Seidel	Linear One-sided Stability of MAT for Weakly Injective Domain
MPI-I-2001-4-003	K. Daubert, W. Heidrich, J. Kautz, J. Dischler, H. Seidel	Efficient Light Transport Using Precomputed Visibility
MPI-I-2001-4-002	H.P.A. Lensch, J. Kautz, M. Goesele, H. Seidel	A Framework for the Acquisition, Processing, Transmission, and Interactive Display of High Quality 3D Models on the Web
MPI-I-2001-4-001	H.P.A. Lensch, J. Kautz, M. Goesele, W. Heidrich, H. Seidel	Image-Based Reconstruction of Spatially Varying Materials
MPI-I-2001-2-006	H. Nivelle, S. Schulz	Proceeding of the Second International Workshop of the Implementation of Logics
MPI-I-2001-2-005	V. Sofronie-Stokkermans	Resolution-based decision procedures for the universal theory of some classes of distributive lattices with operators
MPI-I-2001-2-004	H. de Nivelle	Translation of Resolution Proofs into Higher Order Natural Deduction using Type Theory
MPI-I-2001-2-003	S. Vorobyov	Experiments with Iterative Improvement Algorithms on Completely Unimodel Hypercubes
MPI-I-2001-2-002	P. Maier	A Set-Theoretic Framework for Assume-Guarantee Reasoning
MPI-I-2001-2-001	U. Waldmann	Superposition and Chaining for Totally Ordered Divisible Abelian Groups
MPI-I-2001-1-007	T. Polzin, S. Vahdati	Extending Reduction Techniques for the Steiner Tree Problem: A Combination of Alternative-and Bound-Based Approaches
MPI-I-2001-1-006	T. Polzin, S. Vahdati	Partitioning Techniques for the Steiner Problem

MPI-I-2001-1-005	T. Polzin, S. Vahdati	On Steiner Trees and Minimum Spanning Trees in Hypergraphs
MPI-I-2001-1-004	S. Hert, M. Hoffmann, L. Kettner, S. Pion, M. Seel	An Adaptable and Extensible Geometry Kernel
MPI-I-2001-1-003	M. Seel	Implementation of Planar Nef Polyhedra
MPI-I-2001-1-002	U. Meyer	Directed Single-Source Shortest-Paths in Linear Average-Case Time
MPI-I-2001-1-001	P. Krysta	Approximating Minimum Size 1,2-Connected Networks
MPI-I-2000-4-003	S.W. Choi, H. Seidel	Hyperbolic Hausdorff Distance for Medial Axis Transform
MPI-I-2000-4-002	L.P. Kobbelt, S. Bischoff, K. Kähler, R. Schneider, M. Botsch, C. Rössl, J. Vorsatz	Geometric Modeling Based on Polygonal Meshes
MPI-I-2000-4-001	J. Kautz, W. Heidrich, K. Daubert	Bump Map Shadows for OpenGL Rendering
MPI-I-2000-2-001	F. Eisenbrand	Short Vectors of Planar Lattices Via Continued Fractions
MPI-I-2000-1-005	M. Seel, K. Mehlhorn	Infimaximal Frames: A Technique for Making Lines Look Like Segments
MPI-I-2000-1-004	K. Mehlhorn, S. Schirra	Generalized and improved constructive separation bound for real algebraic expressions
MPI-I-2000-1-003	P. Fatourou	Low-Contention Depth-First Scheduling of Parallel Computations with Synchronization Variables
MPI-I-2000-1-002	R. Beier, J. Sibeyn	A Powerful Heuristic for Telephone Gossiping
MPI-I-2000-1-001	E. Althaus, O. Kohlbacher, H. Lenhof, P. Müller	A branch and cut algorithm for the optimal solution of the side-chain placement problem
MPI-I-1999-4-001	J. Haber, H. Seidel	A Framework for Evaluating the Quality of Lossy Image Compression
MPI-I-1999-3-005	T.A. Henzinger, J. Raskin, P. Schobbens	Axioms for Real-Time Logics
MPI-I-1999-3-004	J. Raskin, P. Schobbens	Proving a conjecture of Andreka on temporal logic
MPI-I-1999-3-003	T.A. Henzinger, J. Raskin, P. Schobbens	Fully Decidable Logics, Automata and Classical Theories for Defining Regular Real-Time Languages
MPI-I-1999-3-002	J. Raskin, P. Schobbens	The Logic of Event Clocks
MPI-I-1999-3-001	S. Vorobyov	New Lower Bounds for the Expressiveness and the Higher-Order Matching Problem in the Simply Typed Lambda Calculus
MPI-I-1999-2-008	A. Bockmayr, F. Eisenbrand	Cutting Planes and the Elementary Closure in Fixed Dimension
MPI-I-1999-2-007	G. Delzanno, J. Raskin	Symbolic Representation of Upward-closed Sets
MPI-I-1999-2-006	A. Nonnengart	A Deductive Model Checking Approach for Hybrid Systems
MPI-I-1999-2-005	J. Wu	Symmetries in Logic Programs
MPI-I-1999-2-004	V. Cortier, H. Ganzinger, F. Jacquemard, M. Veanes	Decidable fragments of simultaneous rigid reachability
MPI-I-1999-2-003	U. Waldmann	Cancellative Superposition Decides the Theory of Divisible Torsion-Free Abelian Groups
MPI-I-1999-2-001	W. Charatonik	Automata on DAG Representations of Finite Trees
MPI-I-1999-1-007	C. Burnikel, K. Mehlhorn, M. Seel	A simple way to recognize a correct Voronoi diagram of line segments
MPI-I-1999-1-006	M. Nissen	Integration of Graph Iterators into LEDA
MPI-I-1999-1-005	J.F. Sibeyn	Ultimate Parallel List Ranking ?
MPI-I-1999-1-004	M. Nissen, K. Weihe	How generic language extensions enable “open-world” desing in Java
MPI-I-1999-1-003	P. Sanders, S. Egner, J. Korst	Fast Concurrent Access to Parallel Disks
MPI-I-1999-1-002	N.P. Boghossian, O. Kohlbacher, H.-. Lenhof	BALL: Biochemical Algorithms Library
MPI-I-1999-1-001	A. Crauser, P. Ferragina	A Theoretical and Experimental Study on the Construction of Suffix Arrays in External Memory