Strict Basic Superposition and Chaining

Errata

Leo Bachmair
Harald Ganzinger

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The report MPI-I-97-2-011 discusses the example

$$\begin{aligned} &\rightarrow a \approx b, a \approx c \\ &\rightarrow b \approx c \\ a \approx b, a \approx c &\rightarrow \end{aligned}$$

where $a \succ b \succ c$, and rightly states that strict superposition cannot derive a contradiction if tautologies are eliminates. We then move on saying that this remains the case when, in addition, negative factoring and negative superposition into smaller sides of disequations is allowed as it is the case for the caclulus ZK by Zhang and Kapur.

Chr. Lynch has pointed out to us that the latter claim is not true. In fact, if one superposes the second clause into the smaller side b of the third clause and factors one obtains $a \not\approx c$, from which a contradiction can be derived by strict superposition with tautology elimination.

To show that ZK is in fact not compatible with the removal of tautologies consider this slightly modified example

$$\begin{aligned} &\rightarrow a \approx b, a \approx c \\ &\rightarrow b \approx c \\ &\rightarrow f(c) \approx c \\ f(a) \approx c, a \approx c &\rightarrow \end{aligned}$$

which is unsatisfiable. Provided we assume an lpo with $f \succ a \succ b \succ c$, in ZK only one inference is possible, deriving a tautology from the first and the last clause.